8.1 Introduction to Functions

Finding the Domain and Range of a Relation

Definition of a Relation

A relation is any set of ordered pairs. The set of all first components of the ordered pairs is called the domain of the relation, and the set of all second components of the ordered pairs is called the range of the relation.

Example 1: Find the domain and range of the relation.

$\{(2002,364.2),\ (2003,424.9),\ (2004,559.7)^{*}(2005,607.1\}$

The ordered pairs above represent the median housing prices in San Diego metro area for the years 2002 through 2005. The first number in each ordered pair is the year and the second is the median house price in the San Diego metro area for that year.

Determining Whether a Relation is a Function

Definition of a Function:

A function is a relation in which each member of the domain corresponds to exactly one member of the range. No two ordered pairs of a function can have the same first component and different second components.

Example 2: Determining Whether a Relation is a Function

To determine whether a relation is a function, determine if any domain element corresponds to more than one range element.

a. $\{(2,6),\ (3,7),\ (4,5),\ (1,9)\}$

b. $\{(2,6),\ (3,7),\ (2,5),\ (1,9)\}$

c. $\{(2,6),\ (3,6),\ (4,5),\ (1,9)\}$

Note: Portions of this document are excerpted from the textbook *Introductory and Intermediate Algebra for College Students* by Robert Blitzer.

Functions as Equations and Function Notation

Functions are often given in terms of equations rather than as sets of ordered pairs. Consider the equation

$$y = \frac{250(3x+5)}{x+25}$$

where the variable x represents time elapsed in years and the variable y represents the elk population at the end of x years. The variable x is called the independent variable and the variable y is called the dependent variable because its value depends on x. Because the equation represents a function, we say that y is a function of x, and we write y = f(x). The notation f(x) represents the range value that corresponds to the domain value of x.

Example 3: Consider the equation $y = \frac{250(3x+5)}{x+25}$ given in the paragraph above.

a. Is y a function of x? Explain.

b. Write the equation in function notation.

c. Find f(3) and interpret the meaning of your result.

Example 4: Find the indicated function value:

a. f(2) if f(x) = 3x + 7

b. $f(-1)$ if $f(x) = 2x^2 - 7x$

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c. $f(q)$ if $f(x)=\dfrac{x}{x^2+1}$

d. $f(a+h)$ if $f(x)=x^2-3x+2$

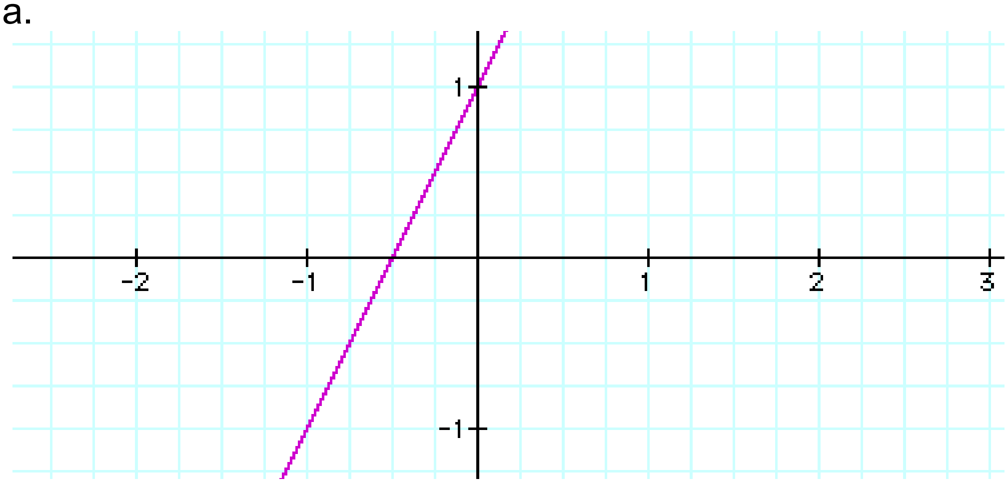
8.2 Graphs of Functions and the Vertical Line Test

The graph of a function or an equation is the graph of its ordered pairs. Not every graph represents a function. If a graph contains two or more different points with the same first coordinate, the graph cannot represent a function. Points that share a common first coordinate are vertically above or below each other. This observation is the basis of a test that is useful in determining if a given graph defines y as a function of x (page 523).

Vertical Line Test for Functions
If any vertical line intersects a graph in more than one point, the graph does not define y as a function of x.

Example 5: Use the vertical line test to identify graphs in which y is a function of x.

a.



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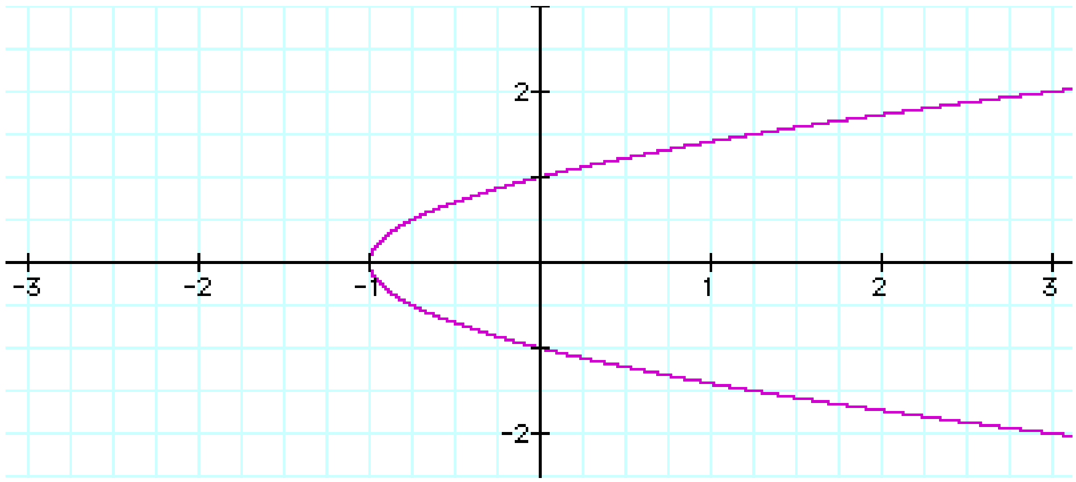

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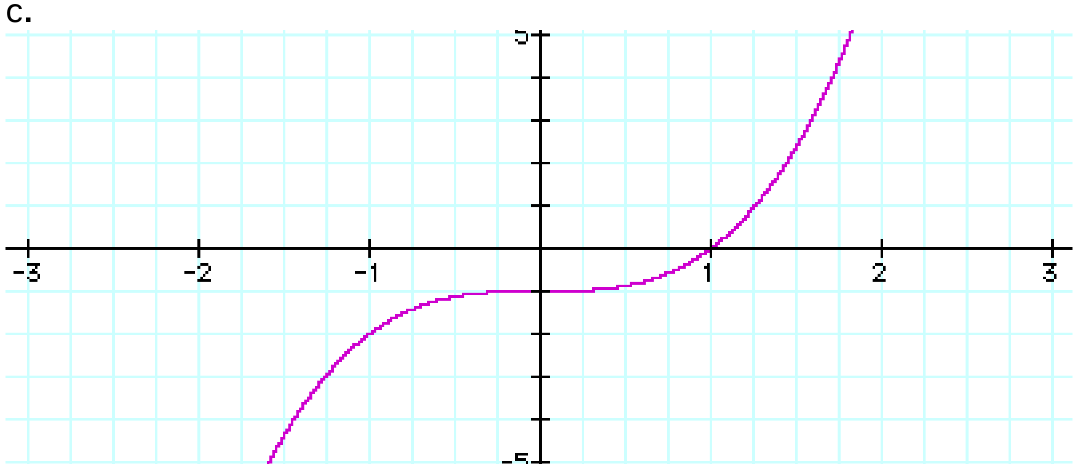

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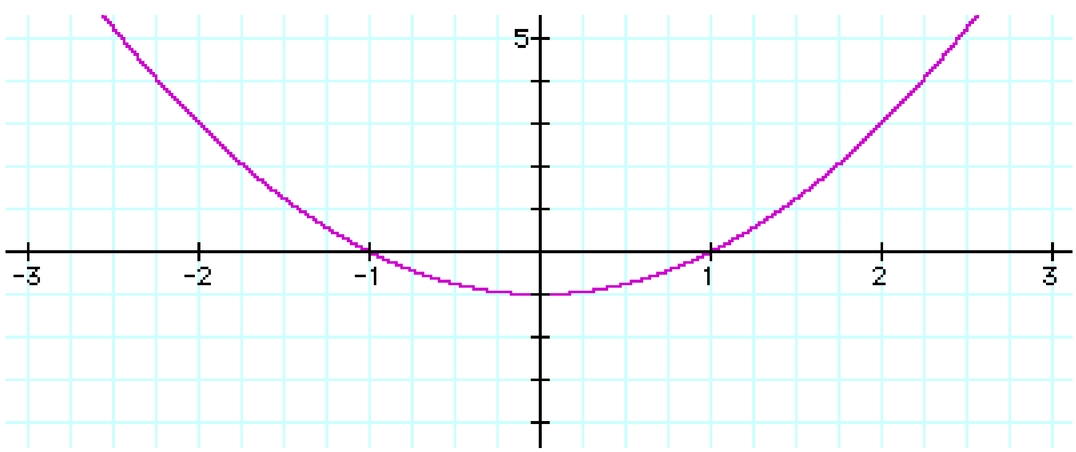

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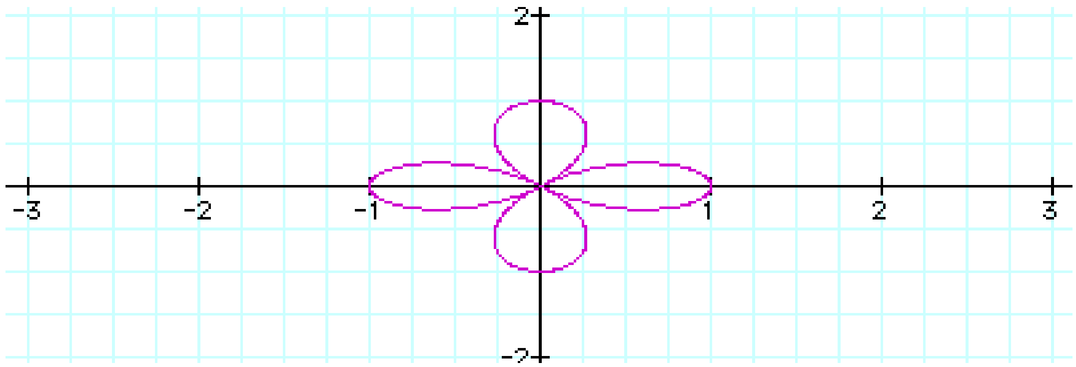
e.



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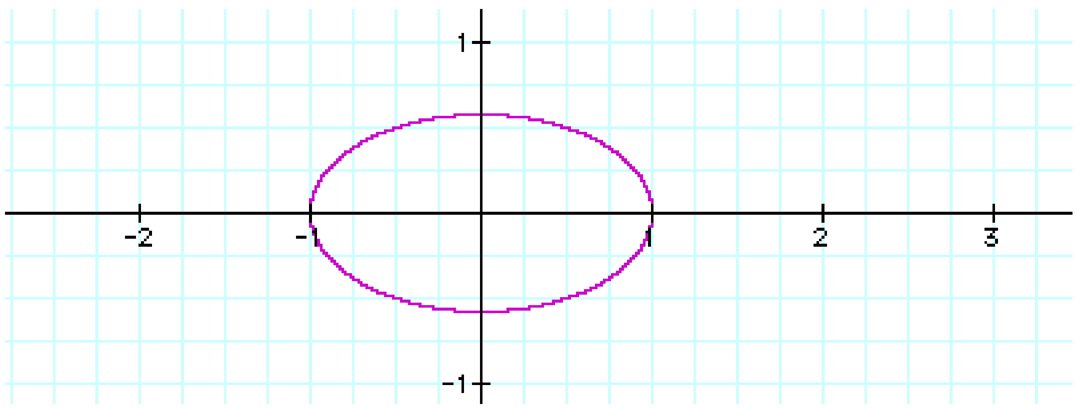


Obtaining Information from Graphs

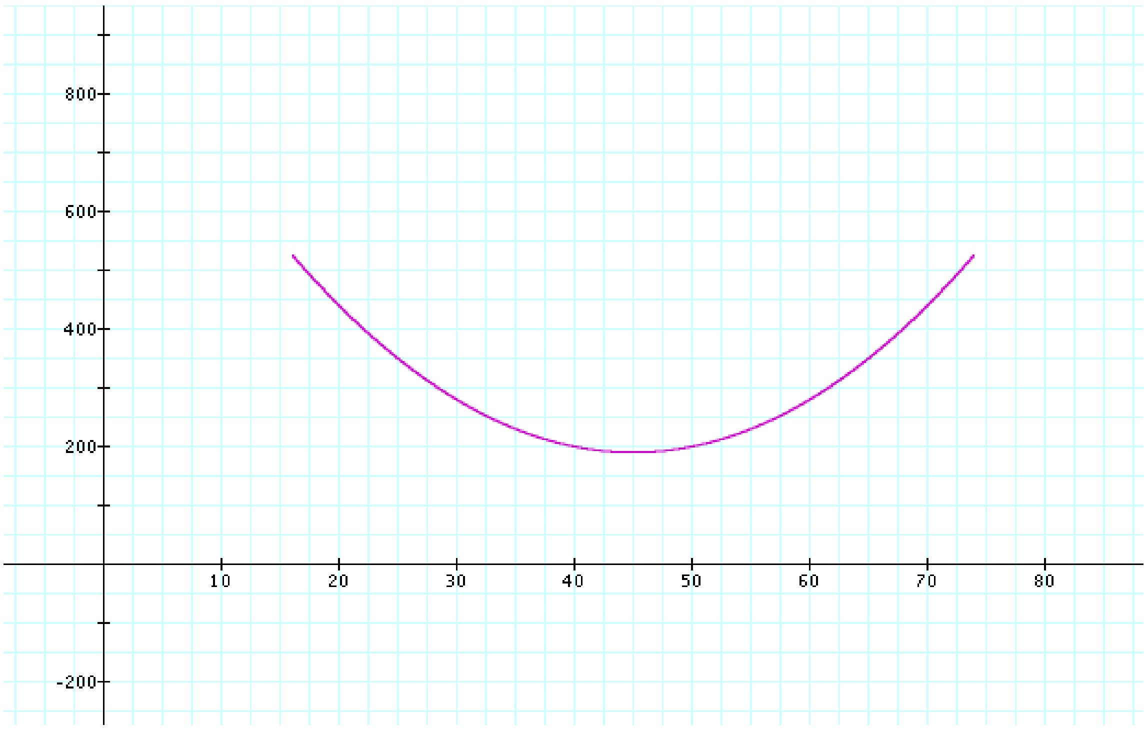
You can obtain information about a function from its graph. At the right or left of a graph, you will often find closed dots, open dots, or arrows.

1. A closed dot indicates that the graph does not extend beyond this point and the point belongs to the graph.
2. An open dot indicates that the graph does not extend beyond this point and the point does not belong to the graph
3. An arrow indicates that the graph extends indefinitely in the direction in which the arrow points.

Example 6: The function

$$f(x) = 0.4x^2 - 36x + 1000$$

models the number of accidents, f(x), per 50 million miles driven as a function of the driver's age, x, in years, where x includes drivers from ages 16 through 74. Use the graph of f, shown below, to answer the questions in a, b and c.



a. Find and interpret f(20).

b. For what value of x does the graph reach its lowest point? Use the equation of f to find the minimum value. Interpret this information in the context of the problem.

c. Estimate the minimum value from the graph.

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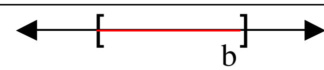
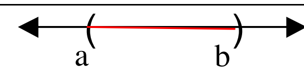
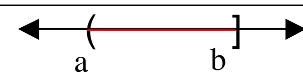
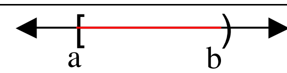
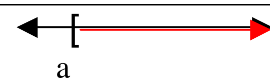
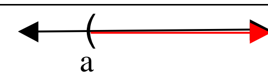
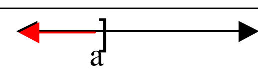
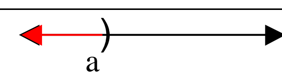
Interval Notation
Domains and ranges are intervals, and interval can be expressed in interval notation, set-builder notation (using inequalities) or graphically on the number line. The following chart shows the different notations. On tests and your homework, you may use interval notation, inequality notation or set-builder notation to depict intervals. **MyMathLab** and **WebAssign often require interval notation.**

Let a and b represent two real numbers with $a < b$.

Type of Interval	Interval Notation	Set-Builder Notation	Graph on the Number Line
Closed Interval	$[a,b]$	$\{x \mid a \le x \le b\}$	
Open Interval	(a,b)	$\{x \mid a < x < b\}$	
Half-Open Interval	$(a,b]$	$\{x \mid a < x \le b\}$	
Half-Open Interval	$[a,b)$	$\{x \mid a \le x < b\}$	
Interval That Is Not Bounded on the Right	$[a,\infty)$	$\{x \mid a \le x < \infty\}$ or $\{x \mid x \ge a\}$	
Interval That Is Not Bounded on the Right	(a,∞)	$\{x \mid a < x < \infty\}$ or $\{x \mid x > a\}$	
Interval That Is Not Bounded on the Right	$(-\infty,a]$	$\{x \mid -\infty < x \le a\}$ or $\{x \mid x \le a\}$	
Interval That Is Not Bounded on the Right	$(-\infty,a)$	$\{x \mid -\infty < x < a\}$ or $\{x / x < a\}$	
Interval That Is Not Bounded on the Right	$(-\infty,\infty)$	$\{x \mid -\infty < x < \infty\}$ or $\{x \mid x \text{ is a real no.}\}$	

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Example 1: Write each inequality in interval notation.

a. $x \geq -3$

b. $5 < x < \infty$

c. $x < 7$

d. $-4 \leq x < \infty$

Example 2: Write each interval in set-builder notation.

a. $[-4,\infty)$

b. $(-\infty,5)$

c. $(-7, -2]$

d. $(-1,4)$

Example 3: Graph each interval on the number line.

a. $[-4,\infty)$

b. $(-\infty,5)$

c. $(-3, -2]$

e. $[-2,2]$

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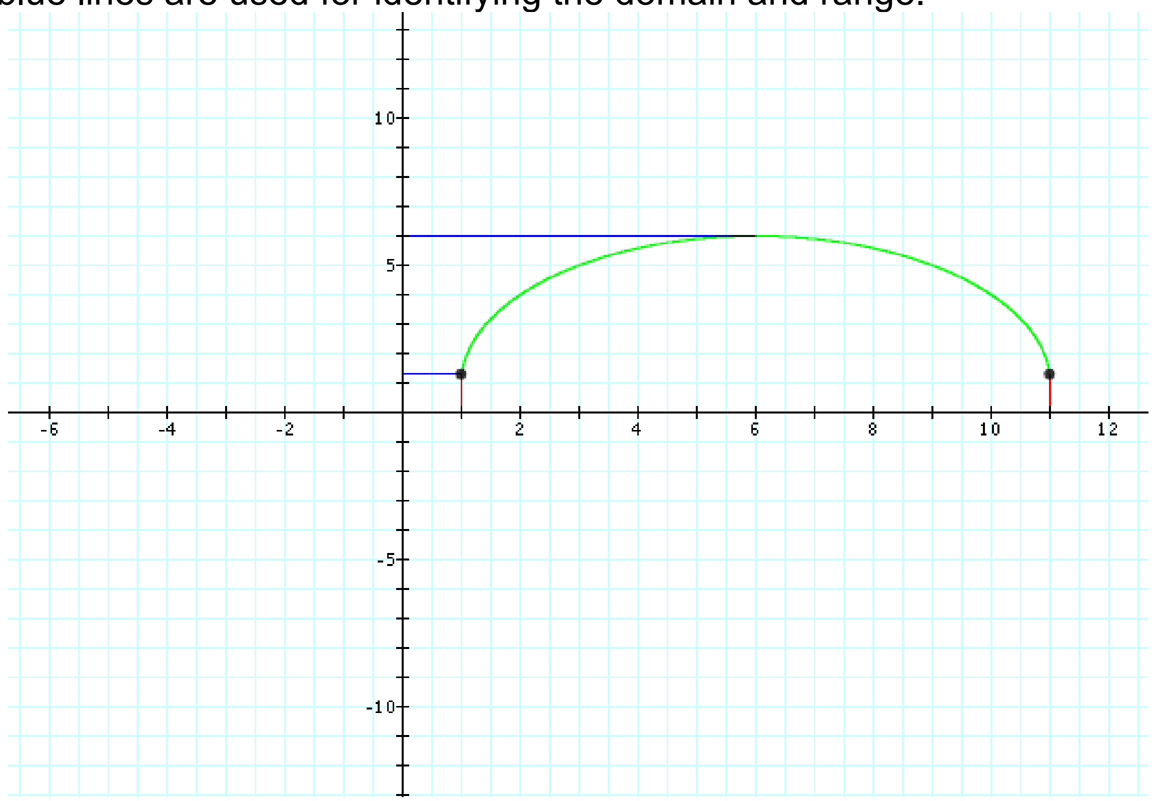
Identifying Domain and Range of a Function from its Graph
To determine the domain and range of a function from its graph:

- Domain: Name the x-axis interval(s) that are traced out when you sweep an imaginary point along the graph and look at the x-coordinates of the points you are tracing.
- Range: Name the y-axis interval(s) that are traced out when you sweep an imaginary point along the graph and look at the y-coordinates of the points you are tracing.

Example 7: Use the graph of each function to identify its domain and range. Give your answers in inequality notation.
a. Note: The graph of the function is given in green. The red and blue lines are used for identifying the domain and range.



Domain: $1 \leq x \leq 11$
Range: $1.3 \leq y \leq 6$

OR in interval notation: Domain: [1,11]
Range: [1.3,6]

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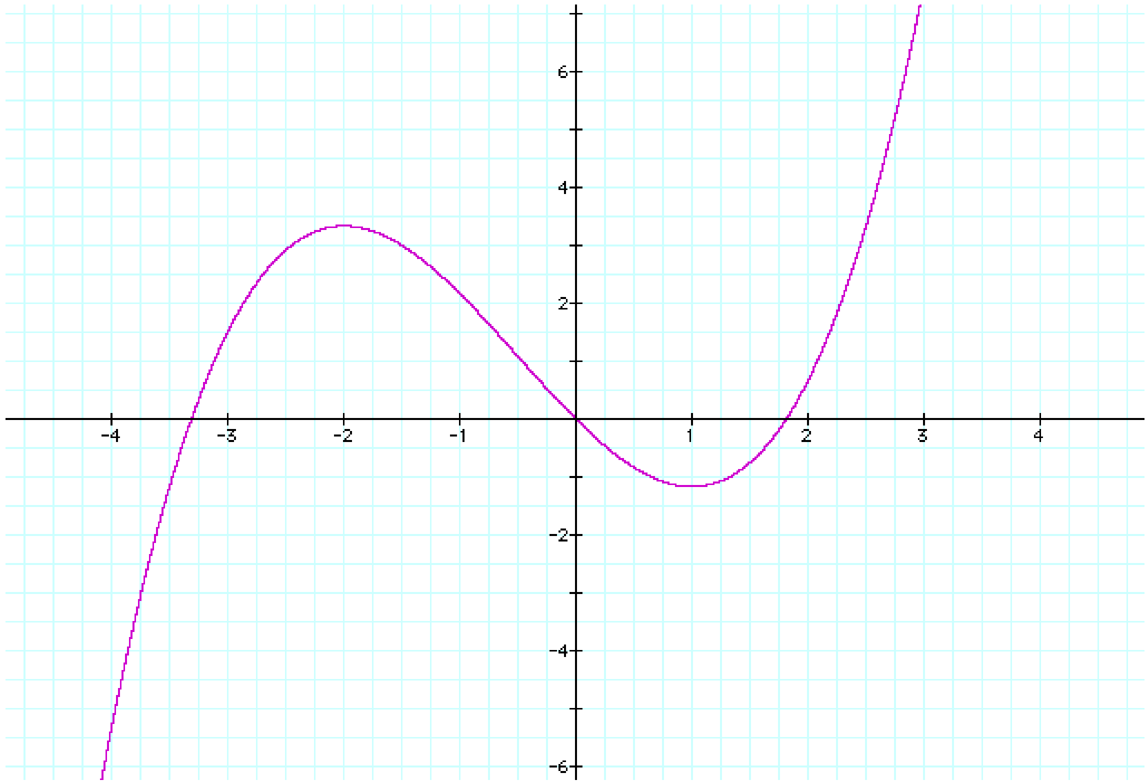
b.



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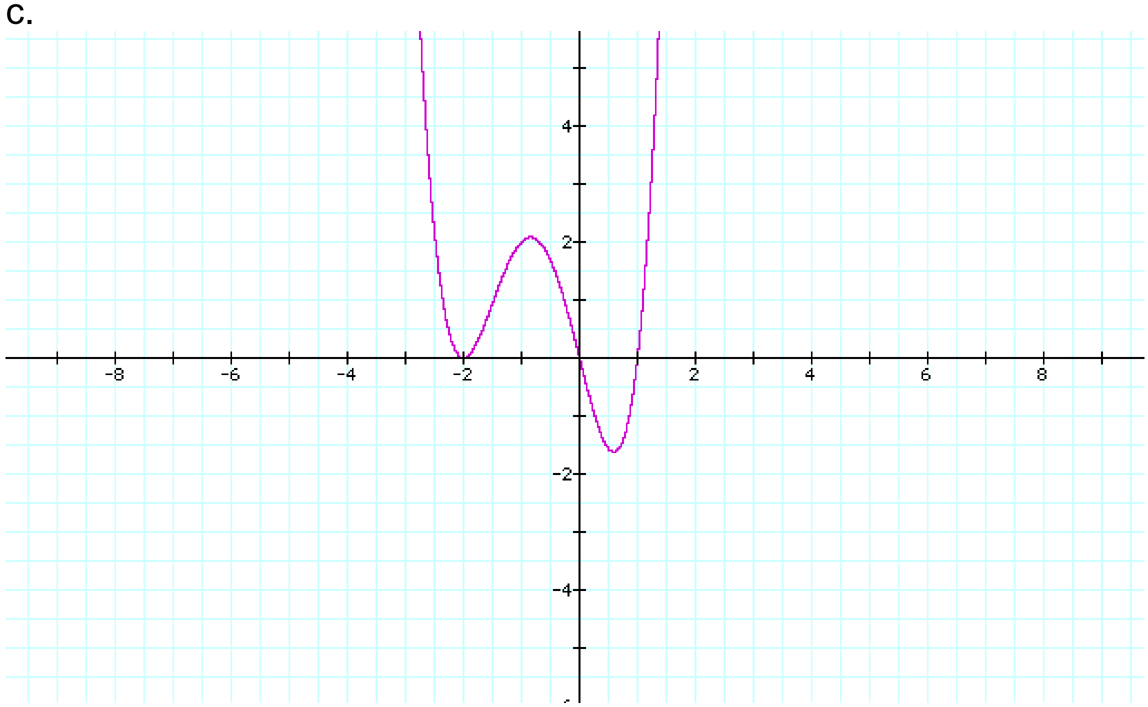

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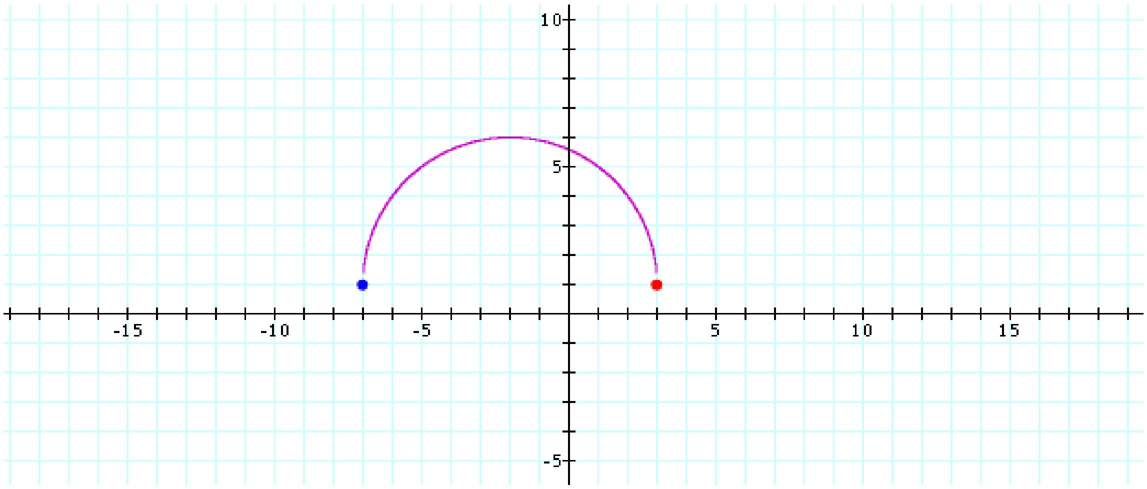
d.



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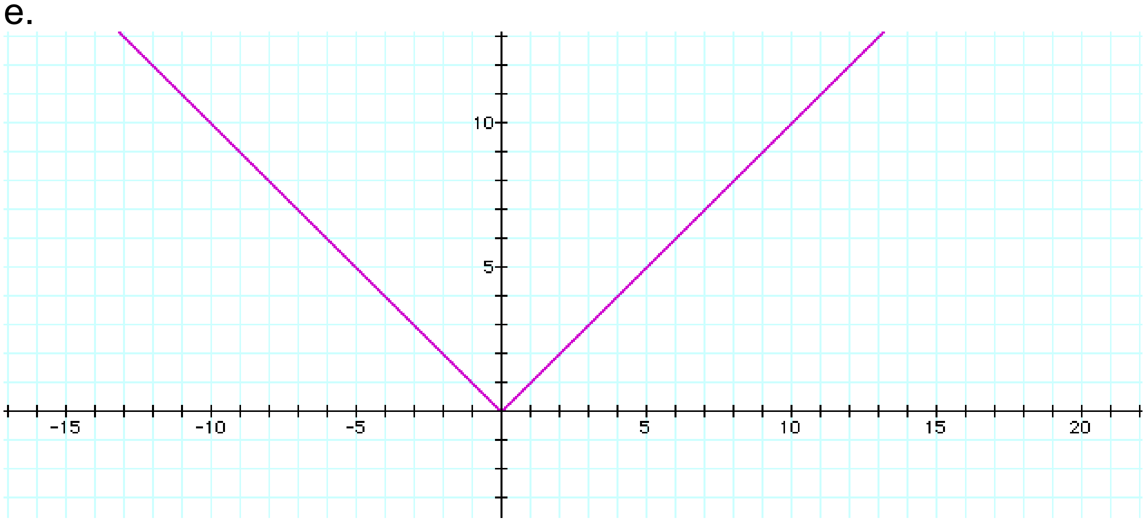

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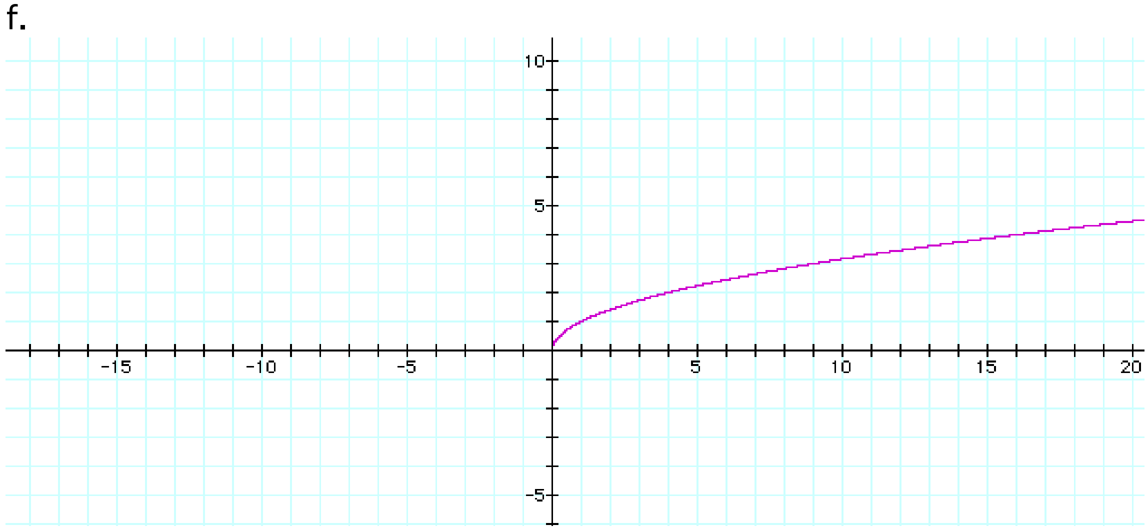

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Answers Section 8.1

Example 1: Domain: {2002, 2003, 2004, 2005)
Range: {364.2, 424.9, 559.7, 607.1}

Example 2:

a. Is a function. Each first element corresponds to exactly one second element.
b. Is not a function. The first element “2” corresponds to two different second elements, “6” and “5”.
c. Is a function. Each first element corresponds to exactly one second element.

Example 3:

a. Y is a function of x. The elk population (y) after x years have elapsed must be unique. It isn’t logical to think that at the end of, say, 5 elapsed years there would be two different numbers for the elk population. Thus each x (number of years elapsed) must correspond to a unique y (elk population at the end of x years).
b. $f(x) = \dfrac{250(3x+5)}{x+25}$
c. f(3) = 125. After three years have elapsed, the elk population is 125.

Answers Section 8.2

Example 4:

a. f(2) = 13
b. f(–1) = 9
c. $f(q) = \dfrac{q}{q^2+1}$
d. $f(a+h) = a^2 + 2ah + h^2 - 3a - 3h + 2$

Example 5:

a. Graph represents a function.
b. Graph does not represent a function.
c. Graph represents a function.
d. Graph represents a function.
e. Graph does not represent a function.
f. Graph does not represent a function.

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Example 6:

a. f(20) = 450 Drivers aged 20 have about 450 accidents per 50 million miles driven.

b. The graph reaches its lowest point for the x-value of 45. Using the function f(45) = 190. Drivers aged 45 have the lowest number of accidents per 50 million miles driven.

c. Estimating from the graph, the minimum value (y value of the lowest point) is about 200.

Example 7:

a. Domain: $\{x \mid 1 \le x \le 11\}$ (or in interval notation: $[1,11]$

Range: $\{y \mid 1.3 \le y \le 6\}$ (or in interval notation: $[1.3,6]$

b. Domain: All real numbers (or in interval notation: $(-\infty,\infty)$

Range: All real numbers (or in interval notation: $(-\infty,\infty)$

c. Domain: All real numbers (or in interval notation: $(-\infty,\infty)$

Range: $\{y \mid -1.6 \le y < \infty\}$ (or in interval notation: $[-1.6,\infty)$)

d. Domain: $\{x \mid -7 \le x \le 3\}$ (or in interval notation: $[-7,3]$

Range: $\{y \mid 1 \le y \le 6\}$ (or in interval notation: $[1,6]$

e. Domain: All real numbers (or in interval notation: $(-\infty,\infty)$

Range: $\{y \mid 0 \le y < \infty\}$ (or in interval notation: $[0,\infty)$)

f. Domain: $\{x \mid 0 \le x < \infty\}$ (or in interval notation: $[0,\infty)$)

Range: $\{y \mid 0 \le y < \infty\}$ (or in interval notation: $[0,\infty)$)